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1986 J. Phys. A: Math. Gen. 19 L447

(http://iopscience.iop.org/0305-4470/19/8/007)

J. Phys. A: Math. Gen. 19 (1986) L447-L450. Printed in Great Britain

LETTER TO THE EDITOR

Cluster aggregation by shortest path travel

S S Manna and B K Chakrabarti

Saha Institute of Nuclear Physics, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India

Received 5 February 1986, in final form 5 March 1986

Abstract. We have studied a new growth process of cluster aggregation. In this process, particles are released randomly from a hypersphere enclosing the cluster and stick to those perimeter sites of the growing cluster which are closest (at shortest distance) to the releasing sites. We find $D=1.62\pm0.09$ for $d=2$ and $D=2.02\pm0.13$ for $d=3$ for the fractal dimensions D of such aggregates in Euclidean dimension d. Using Hentschel's mean-field theory, for generalised cluster aggregations, we argue that these aggregates should have the least fractal dimension (because of maximum screening present in this model) among all the aggregation models in which particles coming from outside form the aggregates. The mean-field values of D predicted by the theory are shown to be comparable with observation.

Study of growing cluster aggregates by non-equilibrium processes has received an intensive thrust after the introduction of the diffusion-limited aggregation (DLA) model by Witten and Sander (1981). In this model a cluster grows around a seed particle and the particles are released one after another from the circumference of a circle enclosing the cluster. One typical particle moves in random walk fashion until it comes to the neighbouring site of the aggregate and becomes a member of the growing cluster. The cluster so formed is a fractal object having self-similarity or scale invariance in all length scales. It is now understood that the diffusion of the incoming particles gives rise to the screening effect which controls the growing process and creates holes of all sizes resulting in a fractal structure. The Hausdorff dimension of these clusters is 1.67 and 2.49 in two and three dimensions respectively (Meakin 1983).

Hentschel (1984) proposed a mean-field theory for these aggregates, based on a Flory-type argument. This is a generalisation of the diffusion-limited aggregation model in which the fractal dimension D of the cluster aggregates depends on both the embedding Euclidean space dimension (d) and the fractal dimension of the walk (d_w) of the incoming particles. Expressing the screening length (l) of the walker in the incipient aggregate in terms of the aggregate radius of gyration and the walk dimensions d_w, and minimising the sum of the elastic (entropic) free energy and the repulsive free energy of the random aggregates of blobs of size l, he obtained

$$D(d, d_w)=\frac{4d_w+d(2d_w-4)+5d^2}{5d_w-4+5d}. \qquad (1)$$

Here we study a new growth process of cluster aggregation. In this process a particle is released from a randomly chosen position on the circumference of the enclosing circle and it is placed at a site on the perimeter of the growing cluster which is at the shortest distance from the releasing position. This distance is measured along the lattice edges and if more than one site on the perimeter is at the shortest distance, one of them is chosen randomly.

We have studied this process of cluster aggregation by the method of computer simulation. The radius of the enclosing circle is always made five lattice spacings greater than the distance from the seed to the most distant particle in the cluster (the maximum radius of the cluster). A position is randomly chosen on the circle and all sites within a square (for three dimensions it is a cube) with the centre at the chosen point on the circle are searched for the perimeter position of the cluster. All sites on the sides of the square are at the same distance from the chosen site on the circle. The distances of the perimeter sites from the chosen site on the circle are measured along the lattice edges. One of these perimeter sites, which are at the same (shortest) distance from the releasing point on the circle, is chosen randomly and the particle is placed at that site (see figure 1).

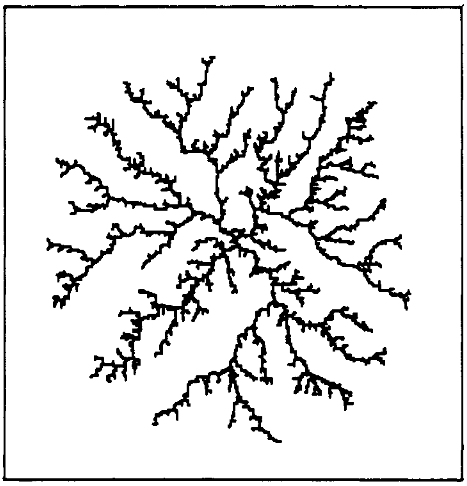

Figure 1. A typical cluster aggregate, containing 5000 particles, grown by shortest path travel on a square lattice.

The Hausdorff dimension D of this cluster is measured using the relation

$$N \propto D^D \tag{2}$$

where N is the number of particles within the Pythagorean distance R from the centre (seed) position of the cluster. The maximum radius of the cluster is divided into fifteen equal parts and numbers of particles within each of these circles are measured. Slopes of the ln N against ln R plots give us the fractal dimension D. In two dimensions six clusters of sizes 15 000 to 20 000 particles give $D(2) = 1.62 \pm 0.09$ (see figure 2). For three dimensions ten clusters are simulated in a $65 \times 65 \times 65$ lattice (simple cubic) until the maximum radius is equal to 32.6 units. On average these clusters contain 675 particles. A plot similar to that for two dimensions gives $D(3) = 2.02 \pm 0.13$ (see figure 3).

Here, the number of perimeter sites accessible to the incoming particle is very few. For these particles therefore, the final positions are almost fixed given the starting (releasing) site. Since the walker may take any number of steps to reach that (those) final (fixed) sites, one can effectively take $d_w \to \infty$ in this case. This is also consistent

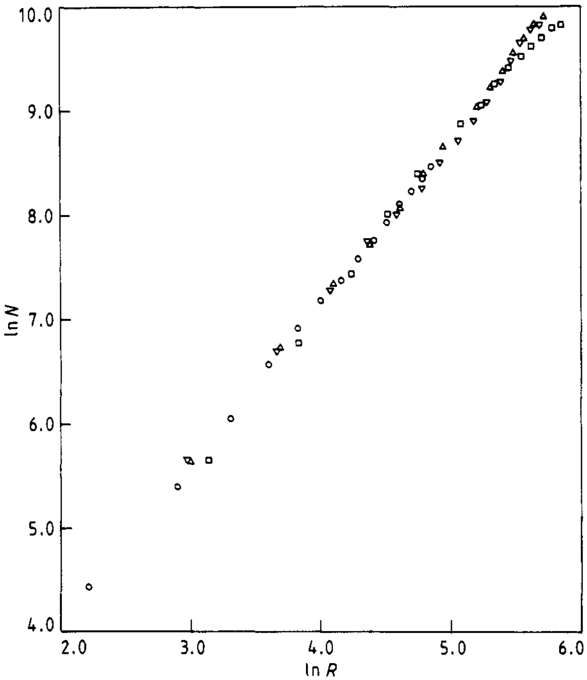

Figure 2. Log-log plot of aggregate mass N against average radius R for the square lattice ($d = 2$) shown for four typical configurations (indicated by four different symbols). Fitting to $N = cR^D$ gives, for an average over 6 configurations, $D = 1.62 \pm 0.09$ and $c = 2.56 \pm 0.62$.

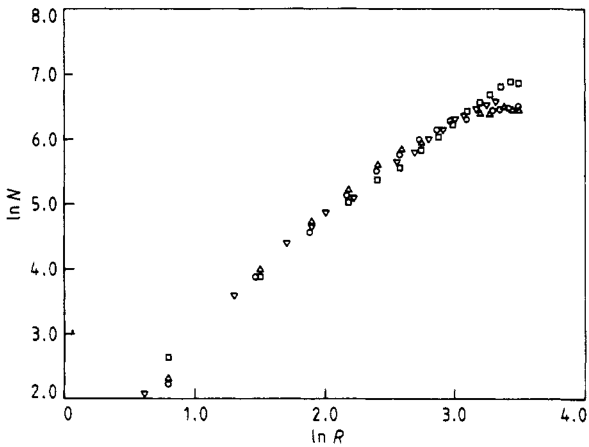

Figure 3. Log-log plot of aggregate mass N against average radius R for the simple cubic lattice ($d = 3$) shown for four typical configurations (indicated by four different symbols). Fitting to $N = cR^D$ gives, for an average over 10 configurations, $D = 2.02 \pm 0.13$ and $c = 2.29 \pm 0.60$.

with the fact that the growth process here is inhibited by maximum possible screening. From equation (1) then we see that in this limit of d_w, $D(2, \infty)$ and $D(3, \infty)$ are 1.6 and 2.0 respectively, which are comparable with our simulation results. It is worth mentioning here that Meakin (1984) has studied DLA on percolation clusters where also the walk dimension was greater than the substrate dimension and the obtained fractal dimensions of the aggregates were compared with similar mean-field estimates (equation (1)). It may also be mentioned that the practice of characterising such cluster aggregates by a single fractal dimensionality may be just as effective, and not necessarily sufficient for very large clusters where anisotropies appear (see e.g. Meakin (1985) for ordinary DLA).

We are grateful to Dr J K Bhattacharjee for useful comments and to VEC (Calcutta) for providing us with the computing facilities.

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